Non-Darcy mixed convection flow along a vertical plate embedded in a non-Newtonian fluid saturated porous medium with surface mass transfer

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Abstract *The present analysis investigates the non-Darcy mixed convection of a non-Newtonian fluid from a vertical isothermal plate embedded in a homogeneous porous medium, in the presence of surface injection or suction. After a suitable coordinate transformation to reduce the complexity of the governing boundary-layer equations, the resulting nonlinear, coupled differential equations were solved with an implicit finite difference scheme. The value of mixed convection parameter* χ *lies between* 0 *and* 1*. In addition, the power-law model is used for non-Newtonian fluids with exponent* $n < 1$ *for pseudoplastic fluids;* $n = 1$ *for Newtonian fluids; and* $n > 1$ *for dilatant fluids. The effects of the mixed-convection parameter* χ*, the power-law viscosity index n, the suction/injection parameter* ξ*, and the non-Darcy parameter* Re* *on the velocity and temperature profiles, and the local Nusselt number are discussed.*

Nomenclature

b = coefficient in the Forchheimer term
d = particle diameter
f = dimensionless stream function
g = gravitation acceleration
h = local heat transfer coefficient
K^* = intrinsic permeability of the porous medium for flow of power-law fluids
n = power-law index of the inelastic non-Newtonian fluid
Nu_x = local Nusselt number
Pe_x = local Peclet numer
Ra_x = local Rayleigh number
Re^* = non-Darcy parameter
T = fluid temperature
T_w = wall temperature
u = streamwise velocity component
U_∞ = free stream velocity
v = normal velocity component
V_o = velocity in the case of the mass transfer
x = axial coordinate
y = normal coordinate

Greek symbols

α = thermal diffusivity
β = volumetric coefficient of thermal expansion
ε = porosity of the saturated porous medium
η = pseudo-similarity variable
μ^* = fluid consistency of the inelastic non-Newtonian power-law fluid
ρ = fluid density
θ = dimensionless temperature
χ = mixed convection parameter
ξ = mass transfer parameter
ψ = stream function

Subscripts

w = conditions at the wall
∞ = conditions at the free stream

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1. Introduction

The study of convective heat transfer from impermeable surfaces embedded in a non-Newtonian fluid saturated porous medium has received much attention in recent years. Such studies have begun to appear only recently because of the importance of changes in heat transfer rates with non-Newtonian flow behavior in the area of ceramic processing, enhanced oil recovery and filtration. On the other hand, a number of industrially important fluids including fossil fuels, which may saturate underground beds, display non-Newtonian behavior. Chen and Chen (1987, 1988) studied the steady state free convection flow of power-law fluids past an isothermal vertical flat plate, horizontal cylinder and sphere embedded in a porous medium. Pascal and Pascal (1989) obtained boundary layer flow of a Herchel-Bulkely fluid along a heated vertical cylinder with constant temperature and constant heat flux. Pascal (1983, 1988) investigated the steady and unsteady flow characteristics of internal flows of non-Newtonian fluids. Nakayama and Koyama (1991) investigated the effects of buoyancy on the flow of non-Newtonian fluids in porous media. More recently, Gorla *et al.* (1997) gave an analysis of mixed convection from a vertical plate in non-Newtonian fluid saturated porous media, taking into account the effect of surface injection or suction.

All the above mentioned studies deal with only the Darcy flow model. However, it is well-known that the Darcy flow model breaks down when the inertia resistance becomes comparable with the viscous resistance. For Newtonian fluids, a squared velocity term in addition to the Darcian velocity term was added to account for this effect which Muskat (1946) called the Forchheimer term. The modified form of the Darcy-Forchheimer equation for non-Newtonian power law fluids has been developed recently by Shenoy (1993). Nakayama and Shenoy (1992) used the Forchheimer extended Darcy model for studying the flow confined within parallel walls subjected to uniform heat flux and immersed in a porous medium saturated with a non-Newtonian power-law fluid. In the present paper, the problem of non-Darcy mixed convection from a vertical plate in non-Newtonian fluid saturated porous media is analyzed. The effect of surface injection or suction is taken into account. Coordinate transformations together with an implicit finite difference method are used to solve the non-similarity problem and to investigate the effects of power-law viscosity index, the suction/injection parameter, the mixed convection parameter on the temperature profiles, as well as the Nusselt number. Gorla *et al.* (1997) analyzed the corresponding problem for the Darcy model. Pascal and Pascal (1997) studied the free convection through a porous medium of a power-law fluid with a yield stress along a vertical surface with lateral mass flux.

2. Analysis

Let us consider mixed convection from a permeable vertical plate embedded in a non-Newtonian fluid saturated porous medium, in the presence of surface injection or suction at a uniform velocity v_o. The coordinate system and flow model are shown in Figure 1.

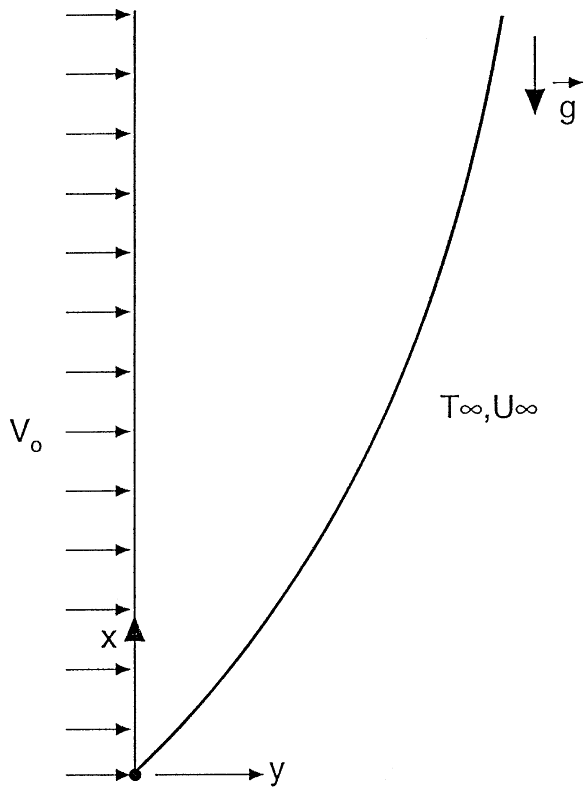

Figure 1.
Coordinate system and flow model

The governing equations under Boussinesq and boundary layer approximation may be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u^n}{\partial y} + \rho \frac{bk^*}{\mu^*}\frac{\partial u^2}{\partial y} = \frac{\rho g \beta\, k^*}{\mu^*}\frac{\partial T}{\partial y} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

In the above equations, u and v are the velocity components, while T is the local temperature of the fluid-saturated porous medium. α is the effective thermal diffusivity of the saturated porous medium, b is an empirical constant associated with the Forchheimer porous inertia term, ρ is the density, μ^* is the consistency index and k^* is the modified permeability for the flow of the non-Newtonian power-law fluid defined as:

$$k^* = \frac{1}{2c_t}\left(\frac{n\varepsilon}{3n+1}\right)^n \left(\frac{50k}{3\varepsilon}\right)^{(n+1)/2} \tag{4a}$$

where

$$k = \frac{\varepsilon^3 d^2}{150(1-\varepsilon)^2} \tag{4b}$$

and

$$c_t = \begin{cases} \frac{25}{12} & \text{according to Christopher and Middleman (1959)} \\ \frac{2}{3}\left(\frac{8n}{9n+3}\right)^n \left(\frac{10n-3}{6n+1}\right)\left(\frac{75}{16}\right)^{3(10n-3)/(10n+11)} & \text{according to Dharmadhikari and Kale (1965)} \end{cases} \tag{4c, d}$$

for $n = 1, c_t = \frac{25}{12}$.

A note may be added on the non-Darcy formulation used in the present paper. A modification of the Darcy-Forchheimer equation is used to include the effect of inertial forces. At a high Raleigh number or in a high porosity medium, there is a departure from Darcy's law and the inertia effects not included in the Darcy model become significant. Ingham (1988) obtained an exact solution for the free convection from a line source in an unbounded non-Darcian porous medium when only the inertia effect is considered, and he shows that the non-Darcian flow would produce a much more peaked temperature profile than that predicted by the Darcian flow. The appropriate boundary conditions are given by

$$\left.\begin{array}{l} y = 0 : v = v_o, T = T_w(const.) \\ y \to \infty : u = u_\infty T = T_\infty \end{array}\right\} \tag{5}$$

The analysis is performed for the buoyancy assisting flow condition. Therefore, for an upward forced flow, we have $T_w \succ T_\infty$ and for downward flow $T_w \prec T_\infty$. The continuity equation is automatically satisfied by defining a stream function $\psi(x, y)$ such that $u = \frac{\partial\psi}{\partial y}$ and $v = -\frac{\partial\psi}{\partial x}$.

Proceeding with the analysis, we define the following transformations

$$\eta = \frac{y}{x}\left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}\right] \qquad \xi = \frac{v_o x}{\alpha}\left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}\right]^{-1}$$

$$\psi = \alpha\left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}\right] f(\xi, \eta) \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\chi = \left[1 + (Ra_x/Pe_x)^{1/2}\right]^{-1} \qquad Pe_x = \frac{u_\infty x}{\alpha}$$

$$Ra_x = \frac{x}{\alpha}\left[\frac{\rho k^* g\beta(T_w - T_\infty)}{\mu^*}\right]^{\frac{1}{n}} \tag{6}$$

On substituting expressions (6) into equations (2) and (3) we have

$$nf'^{n-1}f'' + 2\mathrm{Re}^* f' f'' = (1-\chi)^{2n}\theta' \tag{7}$$

$$\theta'' + \frac{1}{2} f\theta' = \frac{\xi}{2}\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) \tag{8}$$

where

$$\mathrm{Re}^* \frac{\rho\, bk^*}{\mu^*}\left(\frac{\alpha}{x}\right)^{2-n}\left[Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}}\right]^{2(2-n)}$$

The transformed boundary conditions are given by

$$\begin{aligned} &\eta = 0 : f(\xi,0) + \xi\frac{\partial f}{\partial\xi}(\xi,0) = -2\xi,\ \ \theta(\xi,0) = 1 \\ &\eta \to \infty : f'(\xi,\infty) = \chi^2, \theta(\xi,\infty) = 0 \end{aligned} \tag{9}$$

The prime in the previous equations denotes partial differential with respect to η only. We note that $\chi = 0$ corresponds to pure natural convection whereas $\chi = 1$ corresponds to pure forced convection. ξ is positive for injection and negative for suction. In practical applications, it is usually the surface characteristics such as friction factor and Nusselt number that are of importance. Defining the local Nusselt number $Nu_x = \frac{hx}{k_f}$ where $h = q_w/(T_w - T_\infty)$ we have

$$Nu_x = -(Pe_x^{\frac{1}{2}} + Ra_x^{\frac{1}{2}})\theta'(\xi,0) \tag{10}$$

Here, q_w is the heat transfer rate per unit surface area and k_f is the thermal conductivity of the fluid.

3. Numerical scheme

The numerical scheme to solve equations (7) and (8) adopted here is based on a combination of the following concepts:

(a) The boundary conditions for $\eta = \infty$ are replaced by

$$\mathrm{f}'(\xi, \eta_{\max} = \chi^2, \quad \theta(\xi, \eta_{\max}) = 0 \tag{11}$$

where $\eta_{\max}$ is a sufficiently large value of η at which the boundary conditions (9) are satisfied. $\eta_{\max}$ varies with the value of n. In the present work, a value of $\eta_{\max}$ = 25 was checked to be sufficient for free stream behavior.

(b) The two-dimensional domain of interest (ξ, η) is discretized with an equispaced mesh in the ξ-direction and another equispaced mesh in the η-direction.

(c) The partial derivatives with respect to η are evaluated by the second order difference approximation.

(d) Two iteration loops based on the successive substitution are used because of the nonlinearity of the equations.

(e) In each inner iteration loop, the value of ξ is fixed while each of the equations (7) and (8) is solved as a linear second order boundary value problem of ODE on the η-domain. The inner iteration is continued until the nonlinear solution converges with a convergence criterion of 10^{-6} in all cases for the fixed value of ξ.

(f) In the outer iteration loop, the value of ξ is advanced. The derivatives with respect to ξ are updated after every outer iteration step.

In the inner iteration step, the finite difference approximation for equations (7) and (8) is solved as a boundary value problem. We consider equation (7) first. By defining $f = \phi$, equation (7) may be written in the form

$$a_1\phi'' + b_1\phi = S_1 \tag{12}$$

where

$$\begin{aligned} a_1 &= n(\phi')^{n-1} + 2\mathrm{Re}^*\phi \\ b_1 &= 0 \\ S_1 &= (1-\chi)^{2n}\theta' \end{aligned} \tag{13}$$

The coefficients a_1, b_1 and the source term in equation (13) in the inner iteration step are evaluated by using the solution from the previous iteration step. Equation (12) is then transformed to a finite difference equation by applying the central difference approximations to the first and second derivatives. The finite difference equations form a tridiagonal system and can be solved by the tridiagonal solution scheme.

Equation (8) is also written as a second-order boundary value problem similar to equation (12), namely

$$a_2\theta'' + b_2\theta' + c_2\theta = S_2 \tag{14}$$

where

$$\begin{aligned} a_2 &= 1 \\ b_2 &= \frac{1}{2}\phi \\ c_2 &= 0 \\ S_2 &= \frac{1}{2}\xi\left[\phi'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial\phi}{\partial\xi}\right] \end{aligned} \tag{15}$$

The gradients $\frac{\partial\theta}{\partial\xi}$ and $\frac{\partial\phi}{\partial\xi}$ were evaluated to a first-order finite difference approximation using the present value of ξ (unknown) and the previous value of $\xi - \Delta\xi$ (known), with the unknown present value moved to the left-hand side of equation (14).

The numerical results are affected by the number of mesh points in both directions. To obtain accurate results, a mesh sensitivity study was performed. After some trials, in the η-direction 190 mesh points were chosen whereas, in the ξ-direction, 41 mesh points were used. The tolerance for convergence was 10^{-6}. Increasing the mesh points to a larger value led to identical results, up to seven significant decimal places.

4. Results and discussion

In this section, the effects of viscosity index n, mixed convection parameter χ, mass transfer parameter ξ and the non-Darcy parameter Re^* on the mixed convection from vertical surface in porous media are presented. Numerical results will be illustrated and discussed for the values of the viscosity index $0.5 \leq n \leq 2$.

Figures 2 to 7 display results for the velocity and temperature profiles with different values of n, ξ and Re^*. The momentum and thermal boundary layer thicknesses increase with ξ, the surface mass transfer parameter. From these Figures, we conclude that the streamwise velocity at the porous wall decreases with Re^*. The thermal boundary layer thickness increases with Re^*.

Figures 8 to 13 display the variation of Nusselt number with χ for the cases of suction and injection with different value of n and Re^*. It is observed that for selected values of n, ξ and χ the Nusselt number decreases with an increase of Re^*. This is evident from the fact that the inertia effects tend to retard the momentum transport in the boundary layer and thus reduce the heat transfer rate. Also, the inertia term has pronounced effect on the heat transfer rate for higher values of Re^*. From these Figures it is seen that for increasing values of suction parameter ξ, the Nusselt number decreases. Increasing values of the viscosity index also decrease the Nusselt number in both the two cases of suction and injection.

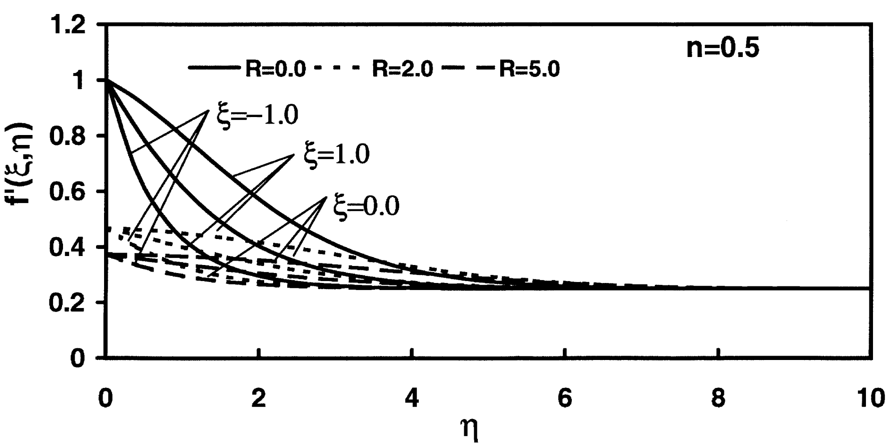

Figure 2.
Velocity profile, $n = 0.5$

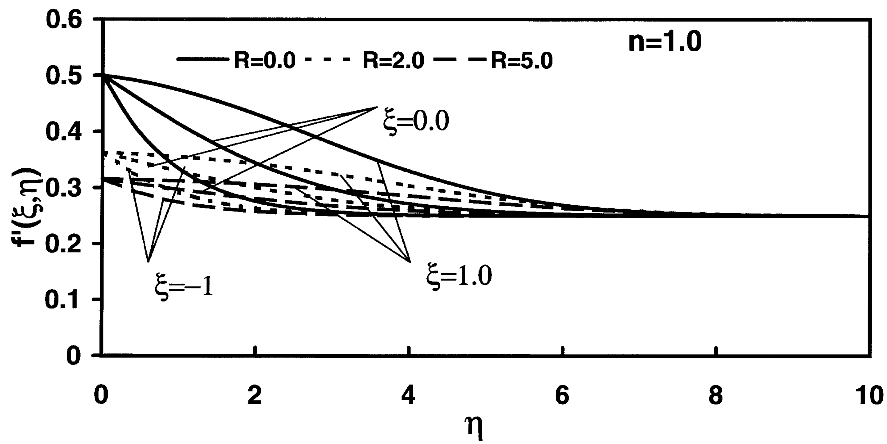

Figure 3.
Velocity profile, $n = 1.0$

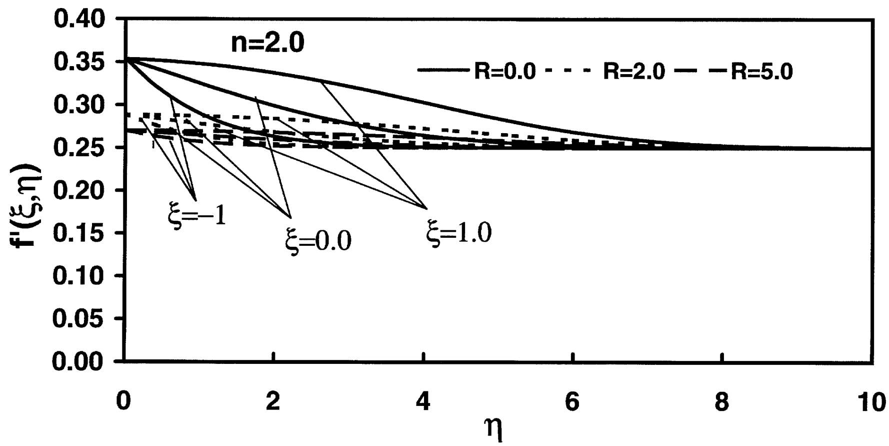

Figure 4.
Velocity profile, $n = 2.0$

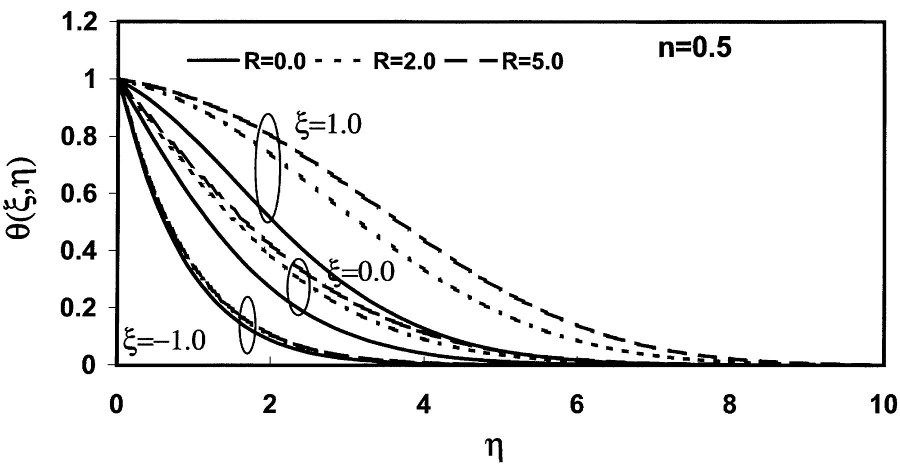

Figure 5.
Temperature profile,
$n = 0.5$

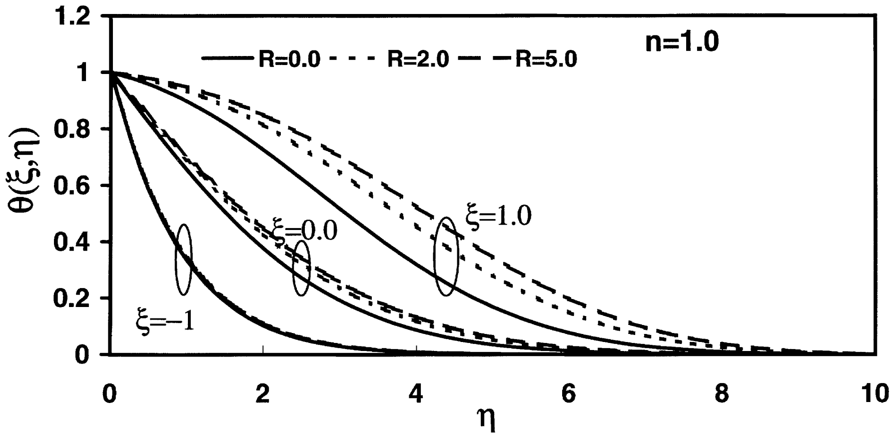

Figure 6.
Temperature profile, $n = 1.0$

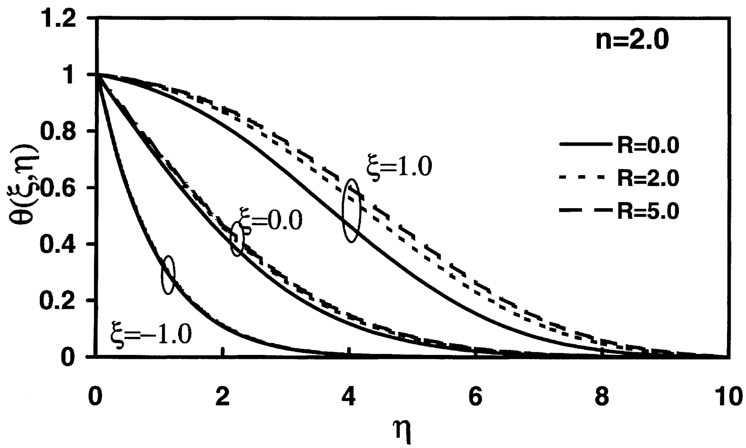

Figure 7.
Temperature profile, $n = 2.0$

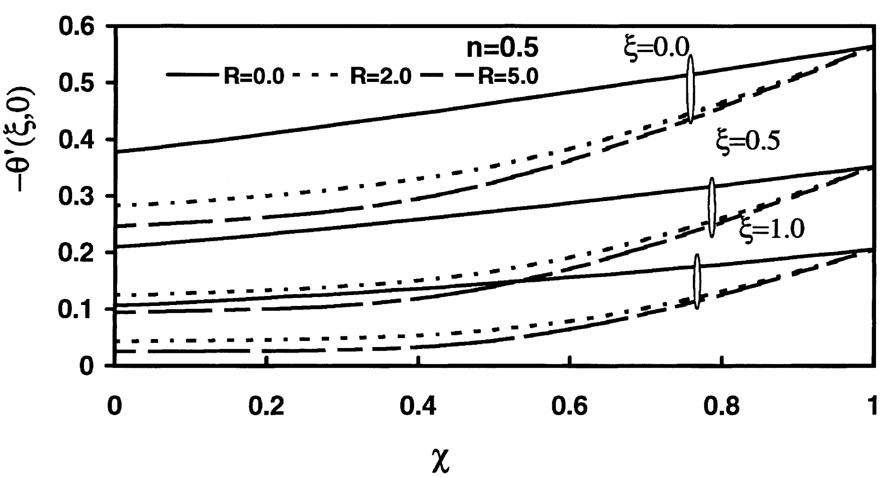

Figure 8.
Variation of Nu with χ for injection, $n = 0.5$

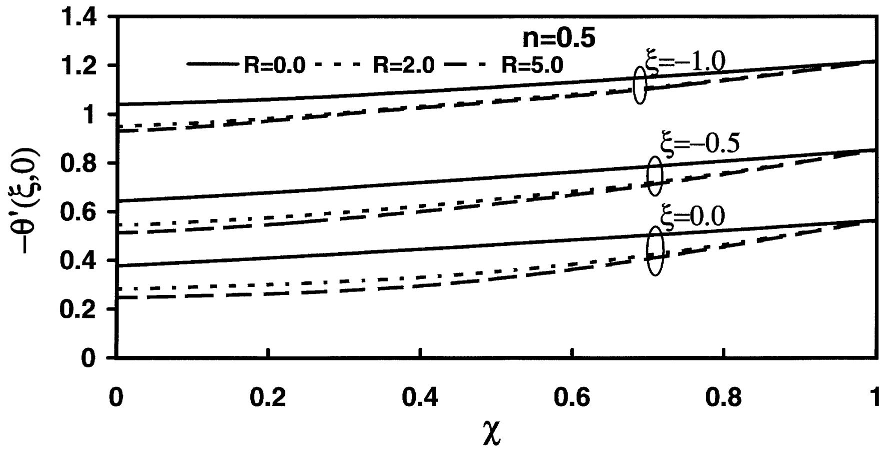

Figure 9.
Variation of Nu with χ for suction, $n = 0.5$

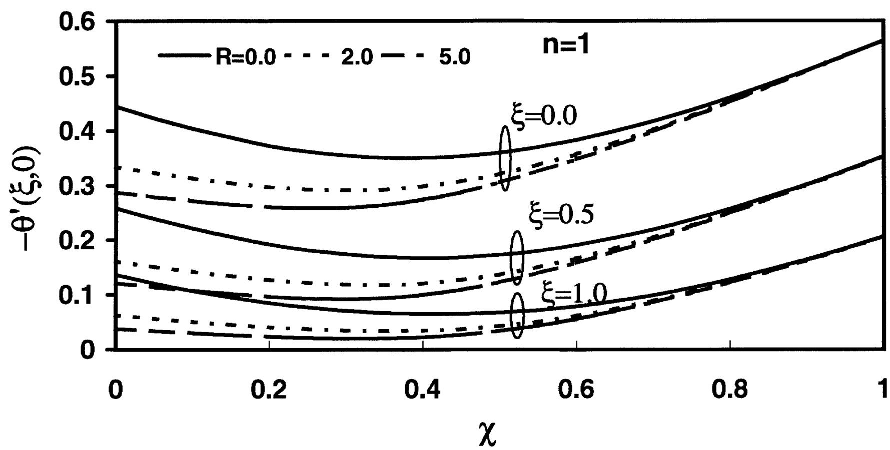

Figure 10.
Variation of Nu with χ for injection, $n = 1.0$

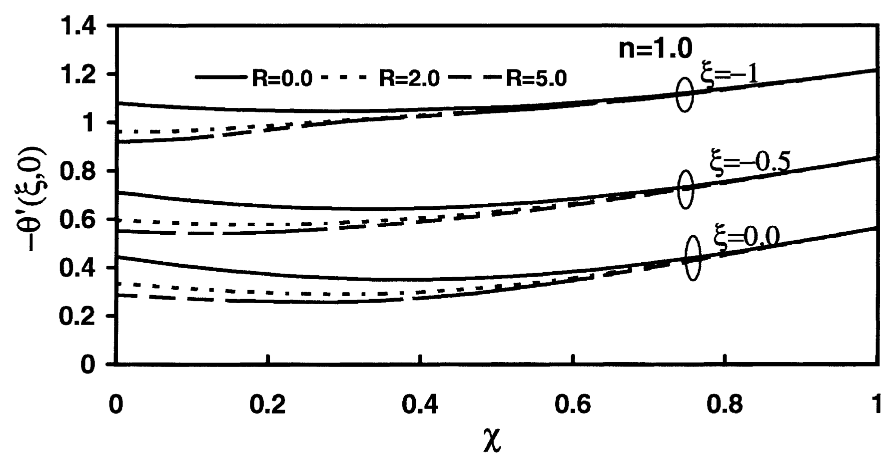

Figure 11.
Variation of Nu with χ for suction, $n = 1.0$

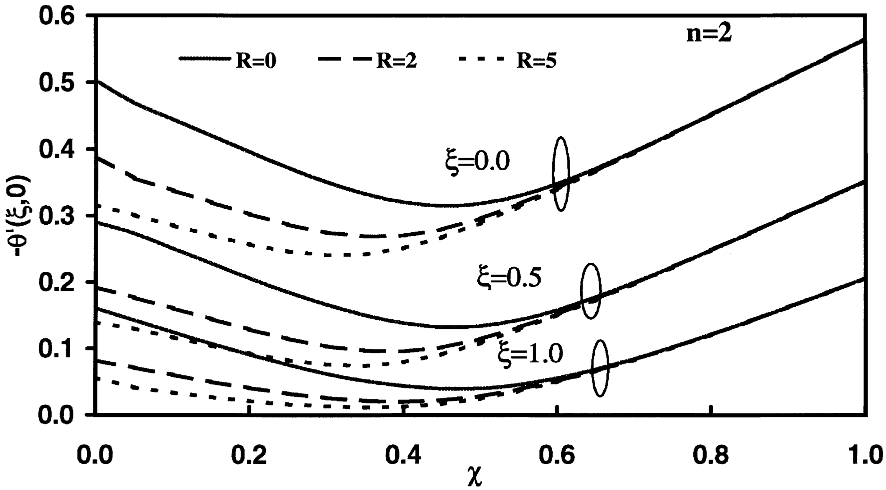

Figure 12. Variation of Nu with χ for injection, n = 2.0

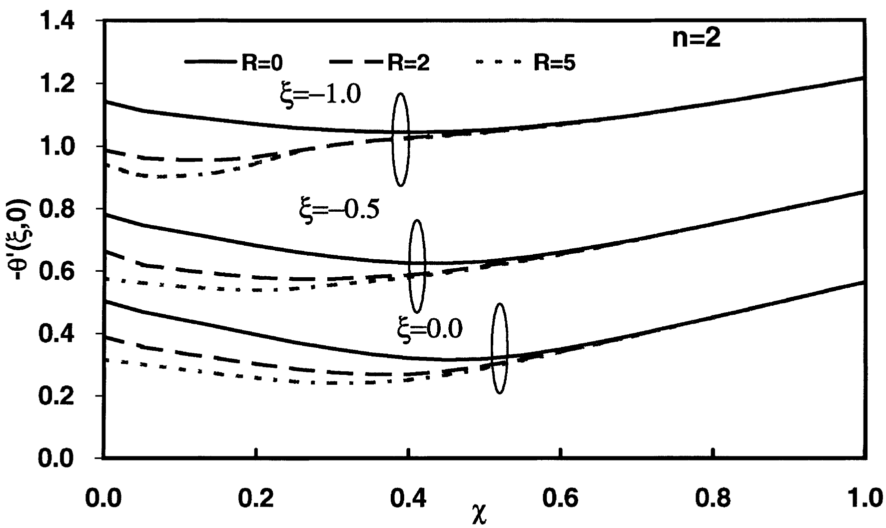

Figure 13. Variation of Nu with χ for suction, n = 2.0

5. Concluding remarks

In this paper, we have presented a boundary layer analysis for the problem of mixed convection from a vertical isothermal surface embedded in a porous medium saturated with Ostwald de-Waele type non-Newtonian fluid. The effect of surface mass transfer is considered. A nonsimilar parameter χ is introduced. Numerical results are presented for the velocity and temperature profiles as well as Nusselt number variation with the combined convection parameter χ. The effects of suction and injection, the viscosity index and the non-Darcy parameter on the surface heat transfer rate have been examined.

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